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Self-consistent description of electrodynamic interaction between two spheres: implications for near-field resonant interactions

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Abstract

The interaction between two spheres with the use of a macroscopic self-consistent approach allowing one to rigorously describe multiple scattering in near-field optics is considered. The implementation of this approach leads to the concept of the effective susceptibility of the scattering system that explicitly relates the incident field to the self-consistent field. We demonstrate that the approach developed can be advantageously used to determine the self-consistent field in systems with simple geometry. Considering two interacting spherical scatterers we show that the self-consistent fields inside the spheres, calculated with and without taking into account multiple scattering, differ noticeably at a resonance.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Progress in near-field optics is caused by development of both experimental technique and mathematical methods of calculations of the near-field structures. Scanning near-field optical microscopy (SNOM) has been successfully used to image with subwavelength resolution numerous surface structures exhibiting different optical contrast mechanisms, such as real and imaginary parts of refractive index, polarization, fluorescence and spectroscopy [1, 2]. Generally speaking, the main principle of SNOM operation is based on electromagnetic interactions in the (illuminated by light) system containing a nanostructured sample and a near-field optical probe. For these interactions to be described properly the self-consistent optical field has to be determined, i.e., the problem of multiple light scattering has to be

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solved. This is a very complicated and challenging problem, and several theoretical self-consistent models have recently been developed [3–5]. On the other hand, the development of physics of composite systems needs adequate theoretical methods to describe the interactions between different particles, which are the basis of the composite materials. This problem is closely related to that of theoretical description of the results of SNOM experiments. It should be noted that there are numerous works in which the problem of interaction between the small particles or between particles and external fields is considered (see, for example, [6–9]). Having no doubts about results of previous studies performed by other authors, in this work we shall consider electrodynamic interaction between two spherical particles with taking into account the inhomogeneity of the local fields inside the particles. To this purpose we shall use the recently developed approach for calculation of the effective susceptibility of a meso-particle [10–12]. We would like to note once more that the present consideration is made with the purpose of using the results for further development of the theory of SNOM.

The self-consistent problem in optics of nano-particles can be formulated by use of the integral Lippmann–Schwinger equation [3]:

$$\vec{E}(\vec{R}, \omega) = \vec{E}^{(0)}(\vec{R}, \omega) - i\omega\mu_0 \int_V d\vec{R}' \overset{\leftrightarrow}{G}(\vec{R}, \vec{R}', \omega) \overset{\leftrightarrow}{\chi}(\vec{R}', \omega) \vec{E}(\vec{R}', \omega), \quad (1)$$

where $\vec{E}(\vec{R}, \omega)$ is the self-consistent field, $\vec{E}^{(0)}(\vec{R}, \omega)$ is the incident field, $\overset{\leftrightarrow}{G}(\vec{R}, \vec{R}', \omega)$ is the Green dyadic and $\overset{\leftrightarrow}{\chi}(\vec{R}, \omega)$ is the local susceptibility (the linear response on the local field) of a particle system with the volume V . The local susceptibility relates the self-consistent (local) field and the current density: $\vec{J}(\vec{R}) = \overset{\leftrightarrow}{\chi}(\vec{R})\vec{E}(\vec{R})$. If one uses the (local) dielectric susceptibility $\overset{\leftrightarrow}{\chi}^{(E)}$, that relates the self-consistent field and the polarization, $\vec{P}(\vec{R}) = (\overset{\leftrightarrow}{\chi}^{(E)}(\vec{R})/4\pi)\vec{E}(\vec{R})$, then the tensor $\overset{\leftrightarrow}{\chi}$ in equation (1) should be replaced by $-i\omega \overset{\leftrightarrow}{\chi}^{(E)}/4\pi$.

The solution of the above implicit integral equation can be found by using discretization of the scattering system under consideration. As a result, the set of linear algebraic equations appears instead of the integral equation:

$$\vec{E}(\vec{R}_\alpha, \omega) = \vec{E}^{(0)}(\vec{R}_\alpha, \omega) - i\omega\mu_0 \times \sum_{i=1}^N \overset{\leftrightarrow}{G}(\vec{R}_\alpha, \vec{R}_i, \omega) \overset{\leftrightarrow}{\chi}(\vec{R}_i, \omega) \vec{E}(\vec{R}_i, \omega), \quad \alpha = 1, 2, \dots, N. \quad (2)$$

Here, it is assumed that the self-consistent field and the susceptibility are constant inside each of N small sub-volumes. The above set of the self-consistent equations can be solved *exactly* with standard procedures of linear algebra. The problem is that the computing time and the number of discretization elements rapidly become unacceptable for extended structures.

One can employ the iteration procedure based on the Born series approach to solve the self-consistent integral equation (1). The idea is to replace the self-consistent field in the integral by the field obtained on the left-hand side of equation (1) during the previous step of the iteration procedure (starting with the incident field). In order to clarify this point let us rewrite equation (1) in the operator form:

$$\vec{E} = \vec{E}^{(0)} + \overset{\leftrightarrow}{J} \cdot \vec{E}, \quad (3)$$

where $\overset{\leftrightarrow}{J}$ denotes the integral operator at the right-hand side of equation (1). Then the Born series expansion of the n th order becomes

$$\vec{E}^n = \vec{E}^{(0)} + \overset{\leftrightarrow}{J} \cdot \vec{E}^{(n-1)} = \vec{E}^{(0)} + \overset{\leftrightarrow}{J} \cdot \vec{E}^{(0)} + (\overset{\leftrightarrow}{J})^2 \cdot \vec{E}^{(0)} + \dots + \overset{\leftrightarrow}{J}^{n-1} \cdot \vec{E}^{(0)}. \quad (4)$$

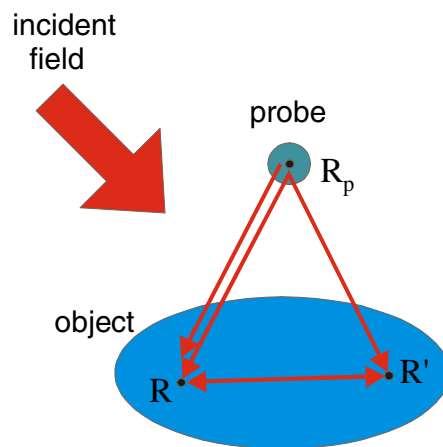


Figure 1. Schematic representation of the considered light scattering processes in the probe–object system.

This expansion is convergent only if the interactions in the system are relatively weak, i.e., if the process of multiple light scattering can be approximated by n th-order scattering. In the case of resonant interactions, the *finite*-order Born approximation cannot be used no matter how many terms in the expansion are taken into account [13]. However, if *all* terms in the Born series expansion are considered, i.e., the *infinite*-order approximation is used, the *exact* solution of the self-consistent equation (1) can be obtained. A similar problem appears in different branches of theoretical physics and is dealt with by using the diagram technique [14, 15].

To obtain an exact solution of the multiple-scattering problem in near-field optics a method was developed in [10–12]. The main idea is to realize exact summation of the infinite series corresponding to the iteration procedure performed for the self-consistent Lippman–Schwinger equation. In the present work the developed method is demonstrated by simulating resonant interactions between two spherical scatterers with multiple scattering being taken into account. We shall call this system a ‘probe–object’ one.

2. Basic equations

Let us consider the problem of multiple scattering in the probe–object system illuminated by a monochromatic incident field at the frequency ω . In many cases, the probe–object coupling can be disregarded [4, 16], but we shall treat rigorously the probe–object interactions (figure 1) so that system resonances, e.g., the probe–object resonance, will also be incorporated in our model. We shall concentrate on very important phenomena such as multiple scattering *inside* the object and the probe and *between* the object and the probe.

2.1. The effective susceptibility of a sphere

To consider these problems we shall use a recently developed scheme allowing one to obtain an analytical solution of a self-consistent problem. The first step of this consideration will be solution of the self-consistent problem for the small sphere (probe) on which the monochromatic field acts.

Let the sphere playing the role of the probe be characterized by a scalar linear response function $\chi^{(P)}(\omega)$ connected with dielectric constant via $\chi^{(P)} = \varepsilon^{(P)} - 1$. If the probe is located in free space with the Green dyadic ($\rho = |\vec{R} - \vec{R}'|$)

$$\overleftrightarrow{G}(\vec{R}, \vec{R}', \omega) = \frac{1}{4\pi} \left[\left(-\frac{1}{\rho} - \frac{ic}{\omega \cdot \rho^2} + \frac{c^2}{\omega^2 \cdot \rho^3} \right) \overleftrightarrow{U} + \left(\frac{1}{\rho} + \frac{3ic}{\omega \cdot \rho^2} - \frac{3c^2}{\omega^2 \cdot \rho^3} \right) \vec{e}_{\vec{R}} \vec{e}_{\vec{R}} \right] e^{i\frac{\omega}{c}\rho}, \quad (5)$$

then the self-consistent field in the probe is determined by the integral equation written in the terms of electrical susceptibility

$$E_i(\vec{R}) = E_i^{(0)}(\vec{R}) - \frac{k_0^2}{4\pi\varepsilon_0} \int_{V_s} d\vec{R}' G_{ij}(\vec{R}, \vec{R}') \chi_{jl}^{(P)} E_l(\vec{R}'), \quad (6)$$

with $k_0 = \omega/c$. Analytical solution of equation (6) can be written via an effective susceptibility [10–12]

$$E_i(\vec{R}) = E_i^{(0)}(\vec{R}) - \frac{k_0^2}{4\pi\varepsilon_0} \int_{V_p} d\vec{R}' G_{ij}(\vec{R}, \vec{R}') X_{jl}^{(P)}(\vec{R}') E_l^{(0)}(\vec{R}'), \quad (7)$$

where the effective susceptibility (the linear response on the external field $\vec{E}^{(0)}(\vec{R}, \omega) = \vec{E}^{(0)}(\omega) e^{i\vec{k}\vec{R}}$) is

$$X_{jl}^{(P)}(\vec{R}, \omega) = \chi_{lk}^{(P)}(\omega) [\delta_{jk} + M_{jk}(\vec{R}, \omega)]^{-1}, \quad (8)$$

with the self-energy part

$$M_{jl}(\vec{R}, \omega) = \frac{k_0^2}{4\pi\varepsilon_0} \int_{V_p} d\vec{R}' G_{jm}(\vec{R}, \vec{R}', \omega) \chi_{ml}^{(P)}(\omega) e^{i\vec{k}(\vec{R}-\vec{R}')}. \quad (9)$$

Let a sphere be subjected to the homogeneous electric field. This means that the phase factor in the integrand of equation (9) have to be replaced with unity. Substituting equation (5) into (9), one has (similar results were obtained in [17, 18])

$$M_{jl}(\vec{R}, \omega) = \frac{\chi^{(P)}}{3} \delta_{jl}. \quad (10)$$

As a result, the effective susceptibility (equation (8)) of the sphere in this special case (the sphere is acted on by the homogeneous field) can be written as

$$X_{(o)}^{(P)} = \frac{\chi^{(P)}}{1 + \chi^{(P)}/3}. \quad (11)$$

Substituting equation (11) into (7) one obtains the well known result that the field inside the sphere is homogeneous and connected with the external field by the following equation [19]

$$\begin{aligned} E_i(\vec{R}, \omega) &= \left[1 - \frac{\chi^{(P)}}{3} \frac{1}{1 + \chi^{(P)}/3} \right] E_i^{(0)}(\omega) = \left[\frac{1}{1 + (\varepsilon^{(P)} - 1)/3} \right] E_i^{(0)}(\omega) \\ &= \frac{3}{\varepsilon^{(P)} + 2} E_i^{(0)}(\omega). \end{aligned} \quad (12)$$

2.2. Self-consistent field in the ‘probe–object’ system

To calculate the self-consistent field in the system consisting of two spherical particles one can introduce the generalized Green dyadic

$$\mathfrak{R}_{ij}(\vec{R}, \vec{R}') = G_{ij}(\vec{R}, \vec{R}') + \overline{G_{il}(\vec{R}, \vec{R}_p)} \cdot G_{lj}(\vec{R}_p, \vec{R}'). \quad (13)$$

The Green dyadic incorporates the *indirect* scattering channel, i.e., field scattering by the small sphere which below will be named the ‘probe’ (the bigger sphere will be named the ‘object’).

The main integral equation for the self-consistent field can be written in the following simple form:

$$E_i(\vec{R}) = E_i^{(0)}(\vec{R}) + \overline{G_{ij}(\vec{R}, \vec{R}_p)} \cdot E_j^{(0)}(\vec{R}_p) - \frac{k_0^2}{4\pi\epsilon_0} \int_{V_s} d\vec{R}' \mathfrak{N}_{ij}(\vec{R}, \vec{R}') \chi_{jl}^{(s)} E_l(\vec{R}'). \quad (14)$$

It should be noted that the main difference between the starting integral equation (1) and the above equation is the presence of the additional Green dyadic (the second term in equation (13)) accounting for the indirect scattering channel related to the field scattering off the probe (see figure 1). In equations (13) and (14) \vec{R}_p points to the probe position and the Green dyadic averaged over the probe volume V_p is introduced:

$$\overline{G_{ij}(\vec{R}, \vec{R}_p)} = -\frac{k_0^2}{4\pi\epsilon_0} \int_{V_p} d\vec{R}' G_{il}(\vec{R}, \vec{R}') X_{lj}^{(p)}(\vec{R}'). \quad (15)$$

The solution of equation (14) can be obtained by the method developed in [10, 11]. As a result, the self-consistent field inside the system under consideration can be represented by introducing the effective susceptibility of the object

$$X_{jl}^{(s)}(\vec{R}, \omega) = \chi_{jk}^{(s)}(\omega) [\delta_{lk} - D_{lk}(\vec{R}, \omega)]^{-1}, \quad (16)$$

with self-energy part

$$D_{lk}(\vec{R}, \omega) = -\frac{k_0^2}{4\pi\epsilon_0} \int_{V_s} d\vec{R}' \mathfrak{N}_{lm}(\vec{R}, \vec{R}', \omega) \chi_{mk}^{(s)}(\omega). \quad (17)$$

Finally, one writes the self-consistent field at the centre of a probe as

$$E_i(\vec{R}_p, \omega) = \left[\delta_{ij} - \frac{k_0^2}{4\pi\epsilon_0} \int_{V_s} d\vec{R}' \mathfrak{N}_{il}(\vec{R}_p, \vec{R}', \omega) X_{lm}^{(s)}(\vec{R}', \omega) I_{mj}(\vec{R}', \omega) \right] E_j^{(0)}(\vec{R}_p, \omega), \quad (18)$$

where the illumination operator

$$I_{ij}(\vec{R}, \omega) = \delta_{ij} + \overline{G_{ij}(\vec{R}, \vec{R}_p, \omega)} \quad (19)$$

is introduced. One should note that the self-consistent field at an arbitrary point inside the system could be written in a form similar to equation (18).

It should be borne in mind that the integral in equation (17) can be improper at $\vec{R} = \vec{R}'$ due to singularity of the Green dyadic at $\vec{R} = \vec{R}'$. This problem can be dealt with by making use of the regular procedure of excluding a ‘principal volume’ when calculating the integral in equation (19) and similar ones [17, 18].

Note that resonances of the ‘probe–object’ system are related to the poles of the effective susceptibility (equation (16)). Since this susceptibility is part of the integrand in the relation (equation (18)) for the self-consistent field (reflecting the non-local nature of the probe–object interactions), the field distribution at a resonance is expected to be smoother than that obtained within the framework of a point-dipole approach [13].

3. The resonance interaction between two spherical particles

Let us consider light scattering in a system of two spherical particles (a probe and an object) to illustrate the approach developed. Let a probe and an object be characterized by their dielectric susceptibilities $\chi_E^{(p)}$ and χ_E , respectively. Such a system is often modelled by using the point-dipole approximation with the polarizabilities of interacting spheres described by the classical Lorentz–Lorenz expression. However, the Lorentz–Lorenz formula is valid only for homogeneous fields, i.e., fields varying slowly across a spherical particle [20]. It is therefore

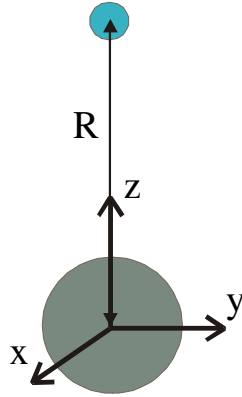


Figure 2. The coordinate system used to describe the self-consistent field in the system of two interacting spheres.

expected that this modelling can lead to erroneous results for closely situated and strongly interacting spherical particles because their interaction results in a strongly inhomogeneous self-consistent field. The fact that it is rather difficult to apply the discretization procedure (equation (2)) to spherical scatterers is an additional reason for choosing this configuration to test the developed approach. In this section we demonstrate that our approach reproduces the results obtained with the use of the point-dipole approximation if we consider homogeneous fields.

It is convenient to choose the coordinate system in which the origin coincides with the centre of the object and the z -axis points toward the probe (figure 2). We have to find the self-consistent field at the site of the probe as a function of the probe–object distance R bearing in mind the possibility of encountering (configurational) resonances [13, 21]. First, we consider sufficiently small spheres and assume that the distance between the particles is so great that one can suppose that they are situated in homogeneous fields. Using only the near-field part of the Green dyadic, one can then obtain for the self-consistent field at the site of the probe the following relation:

$$E_i(\vec{R}_p) = \left\{ 1 + \frac{\chi^{(S)}}{3} R_0^3 G_{ii}^{(d)}(\vec{R}_p, \vec{R}_s) \frac{1}{1 + \frac{\chi^{(S)}}{3} - \frac{\chi^{(P)}}{3} \frac{\chi^{(S)}}{3} R_0^3 r_0^3 G_{ii}^{(d)}(\vec{R}_s, \vec{R}_p) G_{ii}^{(d)}(\vec{R}_p, \vec{R}_s)} \right. \\ \left. \times \left[1 + \frac{\chi^{(P)}}{3} r_0^3 G_{ii}^{(d)}(\vec{R}_s, \vec{R}_p) \right] \right\} E_i^{(0)}(\vec{R}_p), \quad (20)$$

where R_0 and r_0 are the radii of the object and probe, respectively, and $\vec{R}_{(s,p)}$ points to their centres. Besides, the following notation is used:

$$G_{ii}^{(d)}(\vec{R}_s, \vec{R}_p) = G_{ii}^{(d)}(\vec{R}_p, \vec{R}_s) = \begin{cases} -\frac{1}{R^3}, & i = x, y \\ \frac{2}{R^3}, & i = z. \end{cases} \quad (21)$$

Finally, introducing the normalized scattering parameters for both spheres, $\rho_1 = \alpha_1 / (4\pi \varepsilon_0 R^3)$ and $\tilde{\rho}_2 = \tilde{\alpha}_2 / (4\pi \varepsilon_0 R^3)$, where $\alpha_1 = X_{(0)}^{(P)} \varepsilon_0 4\pi r_0^3 / 3$ and $\tilde{\alpha}_2 = \chi_E \varepsilon_0 4\pi R_0^3 / 3$, the above relation (equation (20)) can be rewritten in the form

$$E_i(\vec{R}_p) = \left\{ 1 - \tilde{\rho}_2 \frac{1}{1 + (\chi_E/3) - \rho_1 \tilde{\rho}_2} [1 - \rho_1] \right\} E_i^{(0)}(\vec{R}_p), \quad i = x, y, \quad (22)$$

$$E_z(\vec{R}_p) = \left\{ 1 + 2\tilde{\rho}_2 \frac{1}{1 + (\chi_E/3) - 4\rho_1 \tilde{\rho}_2} [1 + 2\rho_1] \right\} E_z^{(0)}(\vec{R}_p). \quad (23)$$

The above expressions for the self-consistent field at the site of the probe differ from those obtained in the point-dipole approximation [21] due to the term $\chi_E/3$, which describes light scattering (the self-action) inside the particle. The expression $\tilde{\rho}_2(1 + \frac{\chi_E}{3})^{-1}$ can then be transformed into the following relation:

$$\rho_2 = \frac{\alpha_2}{4\pi\epsilon_0 R^3}, \quad (24)$$

with the object polarizability being now expressed by the Lorentz–Lorenz relation,

$$\alpha_2 = 4\pi\epsilon_0 R_0^3 \frac{\epsilon - 1}{\epsilon + 2}, \quad (25)$$

which describes the polarizability of a spherical particle in the homogeneous electrical field [18]. On the other hand, the expression

$$\alpha_1 = X_{(0)}^{(p)} \epsilon_0 4\pi r_0^3 / 3 = 4\pi\epsilon_0 r_0^3 \frac{\chi_E^p}{3 + \chi_E^p} = 4\pi\epsilon_0 r_0^3 \frac{\epsilon_p - 1}{\epsilon_p + 2} \quad (26)$$

describes the polarizability of a spherical probe in a homogeneous electrical field. Then, we have demonstrated that in the frame of the developed approach the well known result of polarizability of the small spheres in the homogeneous field (Lorentz–Lorenz formula) is obtained automatically for interaction between two spheres situated at a large distance from one another.

The self-consistent field at the site of the probe under the above assumptions (the probe and object are small spheres) is given by

$$E_{x,y}(\vec{R}_p) = \left[\frac{1 - \rho_2}{1 - \rho_1 \rho_2} \right] E_{x,y}^{(0)}(\vec{R}_p), \quad E_z(\vec{R}_p) = \left[\frac{1 + 2\rho_2}{1 - 4\rho_1 \rho_2} \right] E_z^{(0)}(\vec{R}_p). \quad (27)$$

These relations coincide with those obtained in the point-dipole approximation [13, 21]. One should note that the expressions (27) were calculated in terms of the self-field (equation (20)), but under the assumption that the external (for each of the two spheres) field varies slowly. It is clear that the latter approximation can be justified only when the particle separation is large enough in comparison with particle sizes. For sufficiently small separations, the field scattered by one of two particles results in an *inhomogeneous* field acting on another particle. This circumstance has to be borne in mind, and the appropriate corrections have to be implemented when calculating the self-consistent field, especially for strongly interacting particles.

4. Numerical results

In this section, we support the above conclusion with the help of numerical simulations. Let us consider the interactions in the system of two spheres described in the previous section (figure 2). Retaining only the near-field part of the Green dyadic in equations (18) and (19), which allows us to describe the resonant interactions in the frame of the quasi-static approximation, the self-consistent field at the site of the probe can be represented by equation (18) with the effective susceptibility of the system given by equation (16). In such a simple configuration, most of the integrals involved in equation (16) can be calculated analytically, and it turns out that the multiple scattering process does not introduce the cross polarization, i.e., the interaction operator is diagonal. This circumstance can easily be accounted for by using symmetry arguments for the configuration considered (figure 2).

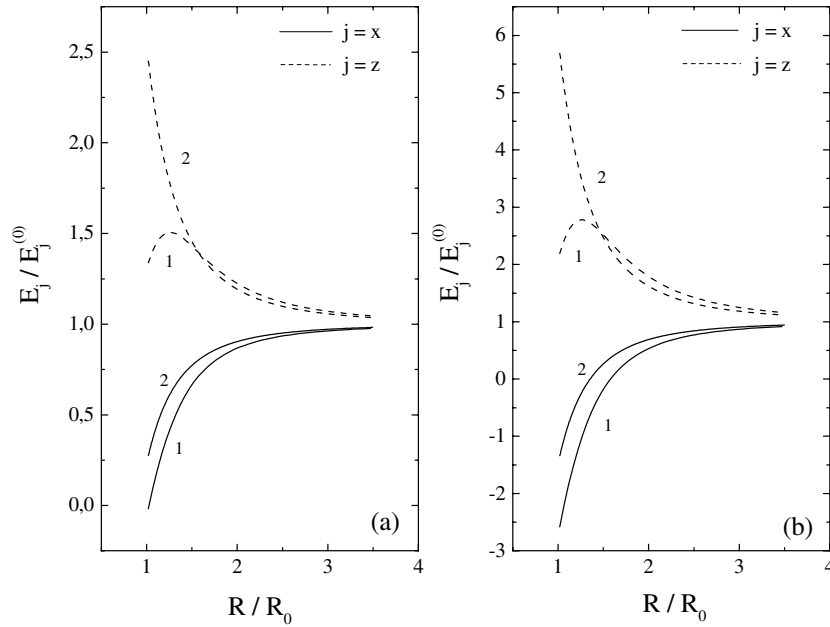


Figure 3. The transverse (solid curve) and longitudinal (dashed curve) normalized components of the self-consistent field at the site of the probe calculated with our approach (curve 1) and by using the point-dipole approximation (curve 2) for the object sphere with different dielectric constants: $\varepsilon = 11$ (a) and $\varepsilon = -4$ (b).

The numerical simulations were carried out in the near-field approximation, because the resonant interactions can still be adequately described [13, 21]. At the same time, this approximation simplifies considerably the integrals involved. Different susceptibilities were used for the object, whereas the dielectric constant of 2.25 ($\chi_E = \varepsilon - 1$) and the radius $r_0 = 0.05R_0$ (R_0 is the radius of the object sphere) were chosen for the probe sphere. The normalized (probe–object) distance dependences of the transverse (x) and longitudinal (z) components of the self-consistent field at the site of the probe were calculated with the developed approach and by using the point-dipole approximation [21] (see figure 3, curves 1 and 2, respectively). It is seen that, at large probe–object distances R , both approaches produce the same results. One can conclude that, for larger R , the total field is more homogeneous and, therefore, the Lorentz–Lorenz approximation is more suitable.

Conversely, for small probe–object distances R , the total field inside the object is noticeably influenced by the probe. Hence, the field perturbations caused by the probe–object interactions are not small and have to be taken into account. The exact solution accounts for these perturbations, whereas in the framework of the point-dipole approximation these corrections are disregarded, that leads to different results. It is also seen that this difference scales with the interaction strength (cf figures 3(a) and (b)). Overall, the obtained results confirm the well known fact that the point-dipole approximation is suitable only for large distances between interacting particles.

Finally, let us consider the resonant probe–object interactions appearing when the dielectric constant of the object approaches -2 [21]. In this case it appears to be reasonable to introduce an imaginary part of the (object) dielectric constant to avoid infinite values of the self-consistent field at the resonance. From the point of view of the point-dipole approximation

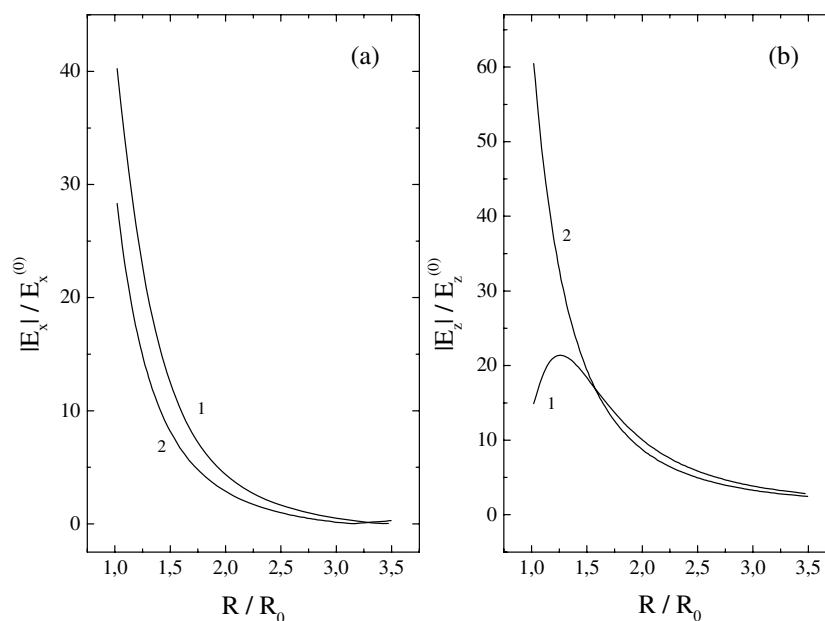


Figure 4. The normalized magnitude of transverse (a) and longitudinal (b) components of the self-consistent field at the site of the probe calculated with our approach (curve 1) and by using the point-dipole approximation (curve 2) for the object sphere with the dielectric constant $\varepsilon = -2.1 + 0.001i$.

and the Lorentz–Lorenz formula, the resonant enhancement of the field scattered by the object is associated with excitation of a spherical plasmon in the sphere. It is seen that both approaches give resonance-like distance dependences of the self-consistent field at the site of the probe (figures 4–6). However, there exist significant differences in these dependences, especially at the probe–object distances close to the resonant values. The origin of these differences has already been mentioned: an inhomogeneous field scattered by the probe not only starts up but also *disturbs* the resonant scattering going on inside the object. Moreover, unlike the point-dipole approximation, our ‘extended sphere—pointlike probe’ model can lead to a set of resonances at different distances R (cf figures 5(b) and 6(b)). The appearance of these resonances can be explained by the fact that the resonance condition $\det(X_{ij}) = 0$, which determines the number of poles in the integrand (equation (18)), might have more than one solution.

5. Concluding remarks

We have developed a self-consistent approach that allows one to obtain an exact solution of the self-consistent problem in near-field optics. The main advantage of the approach developed is that it allows one to introduce the *effective susceptibility* of the scattering system (equation (25)) that *explicitly* relates the incident field to the self-consistent field. This susceptibility contains all system resonances and, for the systems with simple geometry, can be calculated analytically. It can also be modified to include the case when the linear response on the total field (the initial susceptibility) of an object is a non-local function and even the general case of spatially dependent susceptibility. Finally, it should be emphasized that the well known discretization method of solving the self-consistent equation (2) is actually based on the pointlike approximation (for a unit cell), whereas, in our approach, the discretization is used (if

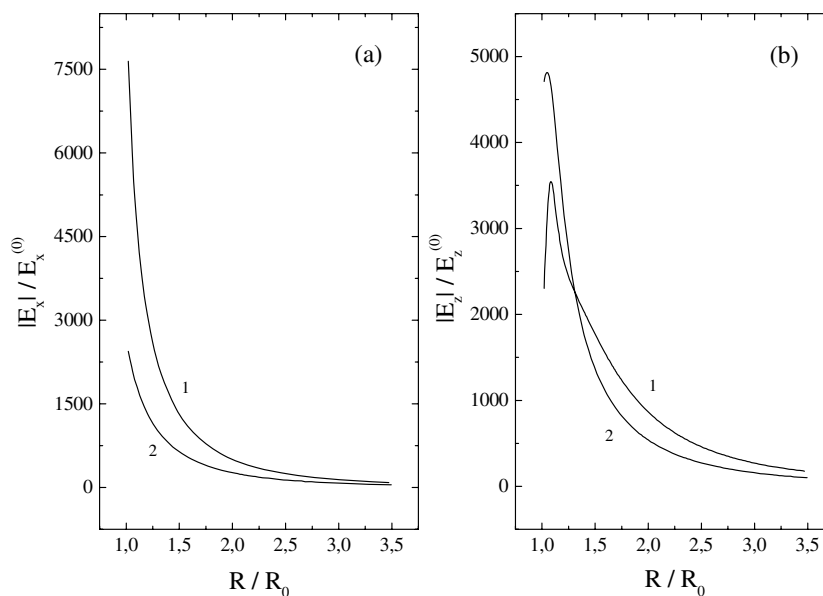


Figure 5. The normalized magnitude of transverse (a) and longitudinal (b) components of the self-consistent field at the site of the probe calculated with our approach (curve 1) and by using the point-dipole approximation (curve 2) for the object sphere with the dielectric constant $\varepsilon = -2.001 + 0.001i$.

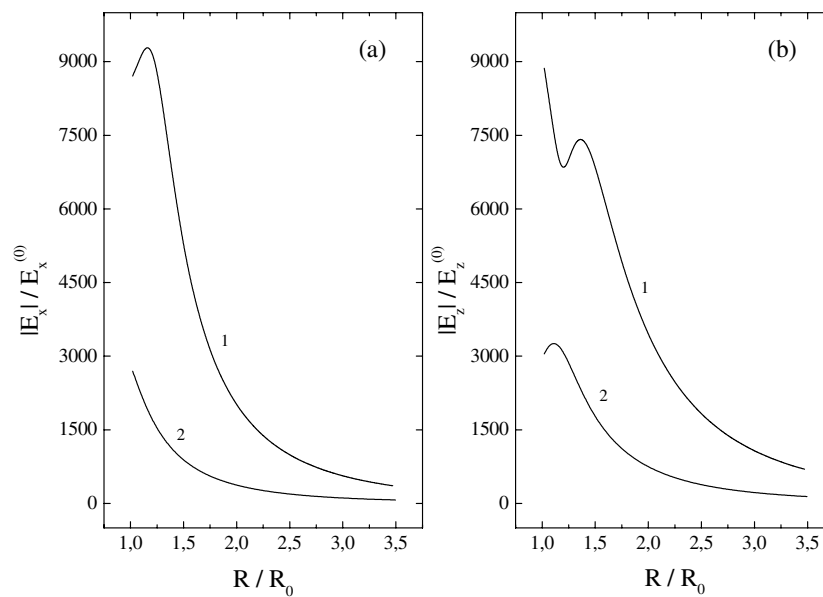


Figure 6. The normalized magnitude of transverse (a) and longitudinal (b) components of the self-consistent field at the site of the probe calculated with our approach (curve 1) and by using the point-dipole approximation (curve 2) for the object sphere with the dielectric constant $\varepsilon = -2.0001 + 0.001i$.

at all) only when calculating the integrals involved (equation (24)). For this reason, numerical simulations with the use of the developed approach can be carried out more accurately than with the use of the direct discretization procedure.

As a first example, we have considered multiple scattering in the system of an extended object sphere and a small spherical probe particle. It has been shown that in the limit of large distances between the probe and the object (the case of homogeneous fields), our approach reproduces the results obtained by using the quasi-point-dipole approximation and the Lorentz–Lorenz formula for the polarizability of spherical particles. We have also found that our approach leads to different values of the self-consistent field at the site of the probe and that this difference is especially pronounced for resonant interactions. We believe therefore that this deviation should be borne in mind when considering multiple scattering in a system of strongly interacting particles, e.g., fractal clusters of nanoparticles [22]. It should be noted that the proposed approach allows one to refrain from involving geometrical renormalization for calculations of the optical properties of small particles [23]. Our approach takes into account the fact that the field distributions inside the particles are inhomogeneous. Therefore, one can do without using a formally introduced intersection of particles, a feature that is a part of the procedure in the frame of the dipole approximation [23]. Our approach can also be used for theoretical studies of ordered arrays of small particles that were found to exhibit various intriguing phenomena, e.g., squeezing the optical near-field zone [24].

It should be noted also that a significant increase of the local field at extremely small ($\sim R_0 + r_0$) distances between the particles may at a certain level be slowed down by nonlinear interactions. However, the study of nonlinear interactions between the particles is out of the frame of this work.

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